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## PROGRESS REPORT

Under

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### Nonlinear Methods Applied to Atmospheric Prediction

by

Richard L. Pfeffer  
Principal Investigator  
Geophysical Fluid Dynamics Institute  
Florida State University  
Tallahassee FL 32306-3017

July 1998

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13. ABSTRACT (Maximum 200 words) Although our research grant was terminated prematurely, we were rather productive in research and publication under AFOSR Grant F49620-96-1-0172. As detailed in section 8 of the Final Report, eleven papers have been published or accepted for publication, three others have been submitted, and six more are in various stages of completion based on research supported entirely or in part by this grant. During the grant period, the principal investigator, Professor Richard Pfeffer, was named <i>Distinguished Research Professor</i> by Florida State University. Also a special issue of the scientific journal <i>Dynamics of Atmospheres and Oceans</i> was dedicated to the scientific and administrative accomplishments of Professor Pfeffer. Significant technology transitions were accomplished, as detailed in Section 6 of the Final Report. The results of our research include (i) the determination of certain limitations on the applicability of the averaging method to problems in which the underlying chaotic attractor (e.g., the Lorenz attractor) consists of motions around two different equilibrium points in planes that are at an angle to each other in phase space. This work is reported in Section 2a of the Final Report. (ii) the determination of the profound influence of meridional shear on the growth and nonlinear life cycles of synoptic-scale atmospheric disturbances, and also on the onset of blocking. Our analytic studies on this subject, completed under this grant, all of which have relevance to the forecast problem, are reported in Section 2b of the Final Report. (iii) the success of a pilot study in statistical weather prediction of precipitation reported in Section 3 of the Final Report (see Fig. 1). (iv) the successful laboratory experimental reproduction of convection cells, like those in the tropics, which are ten times as wide as they are deep, with narrow rising and broad sinking regions (see Section 4, Fig. 2 of the Final Report) and the development of a simple theory that explains this result. (v) the completion of an extensive study of adaptive finite element methods, especially h-p adaptive finite element methods (FEM), for solving partial differential equations, reported in section 5 of the Final Report.					
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## RESEARCH HIGHLIGHTS

Although our research grant was terminated prematurely, we were rather productive in research and publication under AFOSR Grant F49620-96-1-0172. As detailed in section 8, eleven papers have been published or accepted for publication, three others have been submitted, and six more are in various stages of completion based on research supported entirely or in part by this grant. During the grant period, the principal investigator, Professor Richard Pfeffer, was named *Distinguished Research Professor* by Florida State University. Also a special issue of the scientific journal *Dynamics of Atmospheres and Oceans* was dedicated to the scientific and administrative accomplishments of Professor Pfeffer. Significant technology transitions were accomplished, as detailed in Section 6. The results of our research include

(i) the determination of certain limitations on the applicability of the averaging method to problems in which the underlying chaotic attractor (e.g., the Lorenz attractor) consists of motions around two different equilibrium points in planes that are at an angle to each other in phase space. This work is reported in Section 2a.

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(iii) the successful laboratory experimental reproduction of convection cells, like those in the tropics, which are ten times as wide as they are deep, with narrow rising and broad sinking regions (see Section 4, Fig. 2) and the development of a simple theory that explains this result.

(iv) the completion of an extensive study of adaptive finite element methods, especially h-p adaptive finite element methods (FEM), for solving partial differential equations, reported in section 5.

The details of these accomplishments are reported in the following sections.

## DETAILED SUMMARY

### 2. Analytic Studies of Large-Scale Atmospheric Processes

The analytic work is being done by Professors Albert Barcilon and Richard Pfeffer of FSU, Terry Nathan of U.C. Davis and their students and collaborators from various other universities.

**a. Collaboration with Drs. Nicolaenco and Mahalov:** In the early part of the grant period, Drs. Barcilon and Pfeffer had a number of meetings with Drs. Nicolaenco and Mahalov of Arizona State University to discuss possible applications of the averaging technique for obtaining equations governing slow time scales in the presence of both fast and slow modes, and to help Drs. Nicolaenco and Mahalov understand the relevant meteorological problems and equations to which such equations can be applied. At one meeting in Boulder CO, it was agreed that a collaborative effort should be made to extend this work to a hierarchy of more realistic basic states including stratification and internal wind shear. Subsequently, Dr. Barcilon supplied them with the derivation of the primitive equations for a two layer fluid with stratification and shear. We also discussed the application of the averaging method to the Lorenz equations. Having clarified a number of the subtleties of the methodology applied to problems other than the ones already solved by Drs. Nicolaenco and Mahalov, Dr. Pfeffer and students Mike Kirby and Scott Applequist, in consultation with Dr. Barcilon, pursued the solution for the slow time scale in the Lorenz equations as a first step toward applying the methodology to important atmospheric problems. Our motivation for this was that the Lorenz equations have certain important properties in common with the equations governing large-scale mid-latitude atmospheric motions (viz, instability and behavior on both long and short time scales). If we could get something meaningful by applying this methodology to the Lorenz equations, it would seem reasonable to proceed with an attempt to derive equations governing multiple scale interactions among such meteorological waves .

Our original reason for believing that the methodology might work on the Lorenz equations is that the solution of these equations describes fast oscillations which continue for a long time around one equilibrium point before shifting to oscillate around the other equilibrium point. We conceived of the fast oscillations as being the ones over which we could average in order to get equations governing the longer term oscillation between attractor basins.

The exercise has lead us to the conclusion that the averaging methodology is not applicable to the Lorenz equations and, for the same reason, we doubt that it can be applied successfully to problems involving interactions between synoptic scales responsible for cyclogenesis and longer time scales. The simplest way to understand why this is so is to recognize that, while the procedure works on small oscillations from equilibrium in a single basin of attraction, it cannot work in the case of an attractor, such as the Lorenz attractor, which consists of motions around two different equilibrium points in planes that are at an angle to each other in phase space. We can linearize the equations about either of these equilibria, but not both at the same time. As soon as the nonlinear trajectory leaves one basin of attraction, the lin-

earization and the averaging break down. In our annual report last year, we presented the complete mathematical details leading to these conclusions, which we do not repeat here.

**b. Other Analytical Work:** Drs. Barcilon and Nathan, together with their students and colleagues, have undertaken several studies intended to understand the mechanisms involved in rapid growth of large-scale atmospheric disturbances, the profound effects of meridional shear on such growth, the nonlinear response of the atmosphere to external forcing, energy propagation from the tropics to the mid-latitudes and precipitous changes of weather regimes. The results of these studies, all of which have relevance to the forecast problem, are summarized as follows:

In a paper entitled "**Application of Bretherton's interpretation of baroclinic instability in the presence of horizontal shear and compressibility**", published in the special issue of *Dyn. Atmos. Oceans*, 27, 151-160, 1998, (dedicated to Dr. Pfeffer), Drs. W. Blumen and A. Barcilon use normal mode solutions to examine the stability of a non-Boussinesq Eady model with constant horizontal meridional shear in a mid-latitude channel. The effects of compressibility and meridional shear are interpreted in terms of sloping north-south boundaries in an otherwise Boussinesq Eady problem. The focus is on the physical interpretation of these effects in a highly truncated two-meridional-mode solution. It is found that, near the short wave cutoff of Eady, the compressibility and the meridional shear contribute to an effective channel depth which reduces the penetration depth required for instability and these mechanisms therefore tend to destabilize the flow. Yet, compressibility provides a stabilization by attenuating the edge wave's phase speeds which disrupts the phase relationships required to maintain instability.

In a paper entitled "**Non-modal development of baroclinic waves undergoing horizontal shear deformation**", *J. Atmos. Sci.*, 1998, in press, Drs. A. Barcilon and C.H. Bishop use the **non-modal solution** to study the evolution of a basic state containing a constant vertical shear and a smaller constant horizontal shear. They employ "PV-thinking" associated with counter-propagating Rossby waves, propagating as buoyancy waves at the horizontal boundaries, to extract the maximum understanding of the physics of this problem.

The presence of the barotropic shear introduces a time-dependent  $y$ -wave number which in turn makes the total horizontal wave number a function of time. For properly configured initial  $(x, y)$ -wave numbers, the horizontal wave number decreases at first. This decrease induces an increase in the scale of the disturbances and a growth associated with downgradient momentum fluxes. The counter-propagating Rossby waves grow by mutual interactions when properly phase shifted. This growth is due to a downgradient flux of heat.

Drs. Barcilon and Bishop illustrate these ideas on a simple Type-B cyclogenesis. They also compute the nonlinear tendencies in the spirit of a weakly nonlinear analysis. This simple model captures the asymmetry between westward tilts in the vertical of geopotential minima and maxima found in more sophisticated numerical models.

In a paper entitled "**An analysis of the constant shear quasigeostrophic baroclinic-barotropic instability problem**", to be submitted to *J. Atmos. Sci.*, Drs. W.



Blumen and A. Barcilon extend analytically the concepts of a two-level model to two directions: the vertical and the meridional directions. In this study a single finite-difference model of the quasigeostrophic baroclinic-barotropic instability problem is formulated. A zonally directed basic flow, exhibiting constant shear in the vertical and meridional directions, provides the source of linear instability. The model parameters that determine the instability properties are the planetary vorticity  $\beta$  and the Burger number  $B$ , which is a measure of the characteristic vertical to horizontal scale of the basic flow.

The solutions exhibit two separable modes of instability: one associated with long waves (small values of the zonal wavenumber  $k$ ), and the other exhibiting both long- and short-wave cut-offs. The second mode, which requires  $B \neq 1$ , does not have a counterpart in the traditional two layer instability model. It is shown, however, that symmetries and antisymmetries that characterize this model permit an interpretation of the dynamics that is consistent with the interpretation of the conventional two-layer model dynamics that is ingrained in the geophysical fluid dynamics literature. Further, the instability cut-offs may also be explained in terms of Bretherton's analysis of the two-layer model of baroclinic instability. Some possible application to geophysically relevant flows is noted.

In three papers on the events leading up to the onset of blocking in the atmosphere, Drs. Barcilon, Navon and their colleagues and students have examined the role of horizontal shear on the onset of blocking. They find that such shear has a profound influence on these events. In particular, it controls in a subtle way the breaking of planetary scale Rossby waves. It is this breaking which we believe preconditions the atmosphere for block onset. Although not stressed in their published papers, they have found upon further study that the initial perturbations capable of triggering block onset have their largest amplitude in the tropics. These understandings should be of great value to the improved forecast of block onset.

In one paper entitled **"The role of wave breaking, linear instability and PV transports in model block onset"**, *J. Atmos. Sci.*, in press, 1998, M. S. F. V. de Pondeca (Dr. Barcilon's former student), A. Barcilon and X. Zou study model blocks that form in a two-layer isentropic primitive equation model in an effort to understand the mechanisms responsible for the onset of atmospheric blocks. This study concentrates on four blocking events: One of the blocks is present in the control run, while the remaining three are excited by appropriate perturbation of the model's state vector at a pre-selected time when the prevailing flow is classified as zonal. Two phases in the formation of the blocks can be conveniently identified with the parameter calibration chosen in this investigation: The first consists of the formation of cutoff or nearly cutoff cyclones in the upper-layer at low latitudes. The second features a rapid intensification of the upper-layer blocking ridge, accompanied by advection of high-PV beneath it. While the initiation of the first phase may be perceived as far back in time as 6 days before the second phase, the latter occurs on a time scale of 1 to 2 days, giving rise to a well defined blocking pattern.

The authors find that the first phase features the Thorncroft-Hoskins-McIntyre LC1 breaking in the total PV-field that acts as a conditioner of the large scale flow for the second phase to occur. They hypothesize that the second phase consists of (intense) instability of normal mode form, very much as in the theory of barotropic and baroclinic instability of 3-

dimensional basic state flows for the onset of blocks. From a different perspective, based on the concept of interaction between different scales of motion, both phases predominantly involve the transport of synoptic scale potential vorticity by the planetary waves. Planetary-planetary-scale interactions are, however, non-negligible.

In another paper by the same authors, entitled "**An adjoint sensitivity study of the efficacy of modal and non-modal perturbations in causing model block onset**", *J. Atmos. Sci.*, 1998, in press, they use the adjoint sensitivity formalism in conjunction with a blocking index as a response function to assess the impact of normal modes, adjoint modes and regional singular vectors on the prediction of block onset in a two-layer model. They focus on a model flow free of blocks for which a guaranty exists that a block can be excited by appropriate perturbation of the model's state vector at a pre-selected time. This guaranty is provided by searching for "dangerous" synoptic situations with the help of diagnostic tools, including the maximal sensitivity index and the maximal perturbation. The latter is obtained with the help of the adjoint model and defines in the linear context the direction in phase space associated with the largest possible change in the response function. Three case studies of block onset are considered in this work.

The sets of normal modes, adjoint modes and regional singular vectors are computed on instantaneous basic state flows for times preceeding block onset. The authors consider regional singular vectors of the total energy norm and of the streamfunction-squared norm (L2-norm). The sensitivity results presented are for a time window of 3 days and summarize consistent findings for time windows of 2 through 4 days.

The authors evaluate sensitivities by projecting the model's initial condition onto the subspace spanned by the vectors of each set and taking the vector of expansion coefficients as the control variable. They find that some distant normal modes, as ordered by decreasing values of their growth rates, can produce larger changes in the response function than leading normal modes. A similar result holds for the adjoint modes. In contrast, the sets of regional singular vectors contain easily identifiable subsets of structures that have the greatest impact on the response function. The largest changes in the response function are produced by less than the first 20 regional singular vectors, a result found to be true for both norms considered. Also, some of these individual regional singular vectors capture the onset of the block when used as perturbations to the initial condition in a nonlinear model integration. **This result is of importance for ensemble forecasting.** As a reasonable verification for the large impact of the regional singular vectors on the response function, they show that the first five most explosive regional singular vectors of the total energy norm explain over 25% of the norm contained in the maximal perturbation, both at the initial time and at the end of the time window. Remarkably, this value exceeds 60% when they consider regional singular vectors of the L2-norm at the initial time. This increase results from the coincidental choice of the response function defined in terms of an L2-norm. Despite the failure of all individual normal modes in exciting the block, as opposed to adjoint modes and regional singular vectors, they argue that, paradoxically, the concept of normal mode remains a viable tool to explain the dynamical nature of the onset of the block.



In a more recent paper entitled **"Study of Block Onset Using Sensitivity Perturbations in Climatological Flows"** by Zhijin Li (Dr. Navon's graduate student), A. Barcilon and I.M. Navon, submitted to *Mo. Wea. Rev.*, Jan. 1998, the authors found that wind sensitivity perturbations of less than  $10\text{ m s}^{-1}$  and sensitivity forcing of vorticity sources of the order of  $1.5 \times 10^{-10} \text{ s}^{-2}$  are sufficient to excite block onset. When expressed in terms of the vertical component of vorticity, the sensitivity and forcing perturbations for both Pacific and Atlantic blocking display Rossby wave-train structure found primarily on the equatorward flanks of the Pacific and Atlantic jets, near the Philippines and Caribbean, respectively.

In a paper entitled **"Forced baroclinic wave dynamics at minimum critical shear: potential vorticity homogenization, vacillation and equilibration"**, *J. Atmos. Sci.*, 53, 3490–3502, Parlange, M., and T. R. Nathan, 1996 investigated the finite-amplitude dynamics of a slightly dissipative baroclinic wave in a two-layer,  $\beta$ -plane channel model at the point of minimum critical shear. At this point, both the potential vorticity gradient and the Doppler-shifted frequency vanish within the lower layer. Previous studies have shown that, for this parameter setting, both the magnitude of the dissipation and the harmonics of the fundamental wave play important roles in the nonlinear dynamics of the system. In the present study, the response of the nonlinear dynamical system to zonally varying potential vorticity forcing was examined. When the forcing and dissipation are asymptotically small and of equal magnitude, an analytical analysis indicates that the fundamental wave equilibrates to a steady amplitude regardless of the mode being forced. For sufficiently strong forcing the system must be solved numerically, in which case it is shown that, when a harmonic of the fundamental is forced, the system can exhibit one of two dynamical regimes: steady state or oscillatory. The latter can only exist in the presence of forcing. In sharp contrast, directly forcing the fundamental always results in equilibration of the system. In cases where the fundamental wave equilibrates, it is shown that the total potential vorticity (basic state plus disturbance) may homogenize along streamlines of the fundamental wave, leading to strong vortex formation.

In another paper, entitled **"Nonlinear spatial instability in zonally varying baroclinic flow"**, currently in press in *Dyn. Atmos. Oceans*, Dr. Nathan examined the linear and weakly nonlinear dynamics of long, low frequency, spatially growing baroclinic waves embedded in slowly varying zonal flow in a  $\beta$ -plane channel in a continuous model of the atmosphere. For a basic state jet flow possessing a locally unstable region, the nonlinear solution yields a maximum amplitude that is located near the region of maximum baroclinicity and substantially upstream of the maximum amplitude obtained from linear theory. The difference between the linear and nonlinear solutions is due to the time-averaged wave fluxes becoming large enough in the nonlinear problem to stabilize the flow prior to reaching the jet center, where the basic state baroclinicity and locally computed linear growth rate are maximized.

With his student, E. DeFonso, Dr. Nathan has also examined **the connection between the Pacific and Atlantic storm tracks using an idealized nonlinear model** and is preparing a paper for publication in *J. Atmos. Sci.* based on this work. The relationship between wintertime baroclinic regions situated over the eastern shores of Asia and North America is complicated not only by the distance separating the regions, but by the numerous forcings affecting the mean westerly flow. DeFonso and Nathan have examined the extent to

which disturbances that are born from the east Asian jet stream maintain their integrity to affect the eventual growth and maturation of disturbances downstream, especially in the region of the North American jet. In particular, they use a nonlinear, quasigeostrophic potential vorticity conserving model on a mid-latitude beta-plane to examine the dynamical connections between the Pacific and Atlantic storm tracks. A zonally varying basic state forcing is applied to simulate various strengths of the wintertime climatological flows. Preliminary results show that the separation and length of the background jet flows are important in determining the connection between the storm tracks in the model. For example, they find that, for a basic state consisting of two identical jets, the time-mean storm tracks produced in the model may be significantly different depending on the separation distance between the jets.

Together with his student, Long Li, Dr. Nathan also published a paper entitled **"Effects of low frequency tropical forcing on intraseasonal tropical-extratropical interactions"**, *J. Atmos. Sci.*, **54**, 332-346 1996. In this work, a spherical non-divergent barotropic model, linearized about a 300 mb climatological January flow, was used to examine the extratropical response to low frequency tropical forcing. A two-dimensional WKB analysis shows that the energy propagation depends on the sum of three vectors: the basic state wind vector; a vector that is parallel to the absolute vorticity contours; and the local wave vector. The latter two vectors are functions of the slowly varying background flow and the forcing frequency,  $\omega$ . As  $\omega$  decreases, the ray paths approach that of the local wave vector, so that the energy propagates in a direction perpendicular to the wave fronts. The extratropical jet streams have a stronger influence on the long period ( $>30$  d) ray paths than on those of intermediate period ( $\sim 10-30$  d). Except in the jet exit regions, the latitudinal variation of meridional wave scale dominates over the longitudinal variation of zonal wave scale. Global and local energetics calculations show that the energy conversion from the zonally varying basic flow increases as  $\omega$  decreases. The local energetics show that, for the long-period disturbances, both the energy conversion and energy redistribution due to advection and pressure work are significant along the North African-Asian jet stream. The long-period disturbances are less sensitive to the location of the tropical forcing than those of intermediate period. This provides a plausible explanation for atmospheric observations showing that the long-period oscillations tend to be geographically fixed at the exits of the extratropical jet streams, whereas those of intermediate period are zonally mobile wave trains. The long (intermediate) time-scale disturbances dominate in the Northern (Southern) Hemisphere, where the zonal variations in the basic flow are more (less) pronounced.

Drs. Barcilon and Nathan, in collaboration with Dr. Nathan's student, M. Casey, have completed **a study on the effects of seasonal thermal forcing on atmospheric low-frequency variability** and are preparing a manuscript for publication based on this study. Their work is intended to explain differences in the weather patterns observed in the same season during different years. It examines the effects of seasonal thermal forcing on atmospheric low-frequency variability, wave number selection, and the seasonal variation of atmospheric predictability. To address this objective they use a nonlinear, two-layer quasigeostrophic, potential vorticity conserving model on a  $\beta$ -plane channel, with mechanical drag, zonal mean thermal forcing and thermal dissipation. Preliminary results show that the seasonal forcing has a strong effect on the dominant low frequency mode and on the dominant spectral component.

In a paper entitled "**Stochastic dynamics of El Nino-southern oscillations**", to be submitted for publication, Dr. Barcilon and his former student, Professor. B. Wang of the University of Hawaii, have collaborated with Z. Fang to investigate the nature of the highly irregular El Nino-Southern Oscillation (ENSO) cycles using a stochastically forced nonlinear dynamic system that encapsulates the essential physics of ENSO. In this work, they obtain a coupled nonlinear set of two equations controlling the time evolution of the sea-surface temperature anomaly and thermocline anomaly of ENSO. The simplicity of this nonlinear model allows them to bring to bear on this phenomenon an understanding derived from a large class of mathematical studies dealing with unforced/forced nonlinear oscillators.

The model irregular ENSO-line oscillation arises from three different origins – viz., stochastic resonance, coupled nonlinear dynamics, and stochastic transition. When the climatological mean state is stable, stochastic forcing can excite an irregular oscillation by stochastic resonance. When the system is unstable and the coupled dynamics sustain a nonlinear oscillation (stable limit cycle), the stochastic forcing acts to disperse energy for the intrinsic oscillation, generate irregularities, and change the oscillation period. When the ENSO system possesses two non-climatological stable equilibrium states, stochastic forcing can make the system oscillate by a stochastic transition between the two states.

The stochastic response depends not only on nonlinear dynamic regimes of the ENSO system but also on the temporal structure (spectrum) and strength of the stochastic forcing. The intraseasonal noise is shown to be much more effective than synoptic noise in stochastic resonance and in altering the characteristics of the nonlinear oscillation. Stronger forcing yields an enhanced resonant oscillation with a longer period. The basic state annual variation also tends to enhance the resonant oscillation but reduce the oscillation period considerably. On the other hand, increasing the degree of nonlinearity of the ENSO system tends to weaken the resonant response.

The model results suggest that ENSO may arise from multi-mechanisms. Different mechanism may be at work at different phases of ENSO cycles, depending on the basic state and nonlinear dynamics of the system. The atmospheric Madden-Julian Oscillation may provide effective stochastic forcing in stochastic resonance and alter the nonlinear oscillation of the coupled system.

Dr. Nathan in collaboration with his students L. Shellito and Long Li have also investigated "**The effects of El Nino on the propagation of energy from low frequency tropical forcing to midlatitudes**". In a paper to be submitted to *J. Atmos. Sci.* they used a spherical, non-divergent barotropic model, linearized about basic states corresponding to El Nino and La Nina flow regimes, to examine the far field response to localized steady and fluctuating tropical forcing. In particular, they examined the global low frequency response to such forcing during the El Nino and La Nina phases of the Southern Oscillation and compared this with the response obtained for mean January climatological flow conditions. The role of wave guides in determining the path of energy propagation was investigated, along with the differences in mid-latitude responses of the Northern and Southern hemispheres to the tropical forcing. Preliminary results obtained from WKB and local energetics analyses show that the path of energy propagation from low frequency disturbances is less

sensitive to the background flow than the path of energy propagation from intermediate or higher frequency disturbances. For a given tropical forcing, the far field response differs for the climatological, El Nino, and La Nina basic state flow regimes. The location and frequency of the forcing in each of the various basic states plays an important role in determining the path of energy propagation and the influence on mid-latitude low-frequency variability.

### 3. Statistical Weather Prediction

Dr. Pfeffer and his graduate student, Scott Applequist, have been working on the development of nonlinear statistical guidance using the principal components (PCs) corresponding to low order empirical orthogonal functions (EOFs) of multiple predictors to capture the information contained in the synoptic patterns with a minimum number of inputs. They are taking advantage of the wealth of surface data available at three-hourly intervals, in addition to upper air data, and they have been exploring the use of time sequences of predictors to characterize the nonlinearity of the local trajectory of the weather attractor, rather than use predictors at a single time. Their procedures can be considered to be nonlinear extensions of the Perfect Prog approach. Their work during the grant period has been focused on the use of neural networks and analogs.

The work with neural networks has involved an investigation into the ability of a neural network to predict the future states of two simple chaotic systems, the logistic map  $X_{t+1} = 3.8 X_t(1-X_t)$  and the Lorenz attractor. They found that the use of time sequences of predictors improved the prediction. In order to gain insight into the workings of the network, they examined the effect of the nonlinear transfer function and found that it always produces odd degree polynomials of the original input variables. This means that, with typical transfer functions, neural networks are incapable of creating quadratic functions from linear inputs. Therefore, quadratic terms must be specified explicitly as predictors. Accordingly, they are using advections, together with other predictors, and they are using three-hour time sequences of predictors to further capture the nonlinearity of the weather attractor.

In a pilot study of the analog method they sought analogs from a data base consisting of 33 winters of three-hourly surface pressure, temperature and wet bulb temperature measurements at 189 stations over the eastern United States. Empirical orthogonal functions and their principal components were constructed for each variable. The best analogs were determined from the squared difference between the PCs corresponding to the leading EOFs of the three variables at different times. These were then used to predict the precipitation at the selected stations. In the sample prediction shown in Fig. 1 there are 16 stations with precipitation on 15 March 1952 and 17 on 20 March 1968. Eleven of these overlap, yielding an equitable threat score of 0.465. Encouraged by this result they are presently working to improve the methodology by using a combination of the squared difference between two states being a minimum and the spatial correlation being a maximum to determine the best analogs. They also started exploring the value of finding the best analog of each of the trajectories in the ensemble of numerical predictions from the start of the prediction to the verification time. These might then be used to determine the probability of precipitation for different thresholds of precipitation accumulation at each location during the 12 to 36 hours ending at the verification time based on the percent of

analog that gave precipitation in excess of that threshold at that station. The spread among the trajectories can be used as a measure of the uncertainty of the forecast.

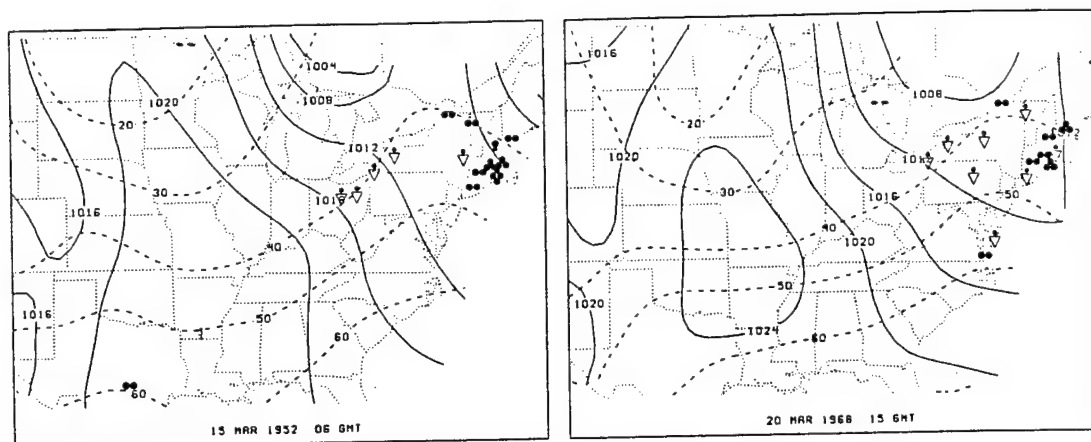


Fig. 1. Sea level pressure (solid; mb), temperature (dashed; deg F) and precipitation distributions for 15 March 1952 06 GMT (left) and 20 March 1968 15 GMT (right). The synoptic chart on the left was determined to be the best analog of the one on the right based on distance in a phase space defined by 45 EOFs (15 each of pressure, temperature and wet bulb temperature).

#### 4. Experimental Studies of Convection

With the support of AFOSR grant F49620-96-1-0172, Professor Ruby Krishnamurti and her former student, Yi. Zhu, have completed and published their work on **"Heat and momentum transport in sheared Rayleigh-Benard convection"**, *Physica D.*, 97, 126-132, 1996 that was previously supported by the Office of Naval Research. In this paper they describe an innovative method of measuring the vertical flux of horizontal momentum in sheared convection. A cylindrical annulus of water uniformly heated below and cooled above was subjected to an imposed shear by driving the bottom boundary so that it rotated steadily around the vertical axis of symmetry. The top boundary was stationary and formed part of a torsion balance. It was suspended from the laboratory ceiling by a long torsion wire. The torque exerted upon the top boundary by the fluid was measured by the twist of the torsion wire. From the angle of twist, the vertical flux of horizontal momentum was determined. The heat flux as well as the momentum flux were measured at each fixed value of Rayleigh number  $Ra$  and Reynolds number  $Re$ , where  $Ra = g \alpha \Delta T d^3 / \kappa \nu$ ,  $Re = \Delta U d / \nu$ ,  $g$  is the acceleration of gravity,  $\alpha$  the thermal expansion coefficient,  $\kappa$  the thermal diffusivity,  $\nu$  the kinematic viscosity,  $d$  the fluid layer depth,  $\Delta T$  the temperature difference and  $\Delta U$  the speed difference between bottom and top boundaries. One of the most striking results they found was that, at Rayleigh number near  $3 \times 10^7$  to  $5 \times 10^7$ , depending upon  $Re$ , the dimensionless momentum flux  $Mo$  ceases its increasing trend and begins to decrease with increasing  $Ra$ . This may be described by saying that the effective viscosity decreases with further increasing of  $Ra$ . However,  $Mo$  always remained greater than unity in the range investigated.



Dr. Krishnamurti's most important new results under the present grant were (i) producing, in the laboratory, convection cells which are ten times as wide as they are deep with narrow rising and broad sinking regions (see Fig. 2) and (ii) developing a simple theory that explains this result.

It has been known for decades that, in Rayleigh-Benard convection, with the fluid layer heated below and cooled above, cellular convection occurs with cells of width very nearly equal to their depth, with areas of rising flow equal to areas of sinking. This has been confirmed in theory and experiment. Furthermore, many variations of the basic problem have been studied which predict some degree of cell broadening, perhaps a few percent. In the Earth's atmosphere, however, cellular convection, as seen in satellite cloud photos, often shows a width to depth ratio on the order of ten. Thus it was with some excitement that Dr. Krishnamurti has been studying her new laboratory results. The experiments, conducted in a **stably** stratified fluid layer, **heated above and cooled below**, with selective internal heating, are laboratory models of atmospheric conditional instability. The following is a brief description.

When there is internal heating in a fluid layer, convection can occur even if the static state is one of stable stratification. Dr. Krishnamurti has been investigating through laboratory experiments such a stably stratified layer of water which is heated above and cooled below. The water contains in dilute solution thymol blue (a pH indicator), which normally colors the water orange. It turns yellow if the pH is low, blue if it is high. A small DC voltage is applied across the layer by using the bottom boundary as the positive electrode and the top boundary as the negative electrode. The hydroxyl ions formed near the bottom boundary cause the orange fluid to turn blue with no accompanying change in density. The fluid layer is uniformly and steadily illuminated from above with light from a sodium vapor lamp. This radiation travels with negligible absorption through the orange fluid but is strongly absorbed by the blue fluid. The resulting warming of the blue fluid can lead to convective instability, with the blue fluid rising into the warm upper layers and continuing to penetrate them as long as it remains blue, and as long as the radiative heating is sufficient to exceed the higher ambient temperatures above. This radiative heating occurs *only in the blue rising flow*. The sinking fluid is orange and is not heated. Dr. Krishnamurti has found that with a strongly stably stratified layer, convective plumes are unable to penetrate far and they remain shallow. Polygonal cells occur whose widths are 8 to 10 times their depth. In this case, the rising region is narrow and the sinking region is broad, so that downward vertical velocities are correspondingly small. In this way the downwards-forced warm fluid has time to cool by conduction to the cold boundary. However, for a weakly stratified layer, plumes grow tall and furthermore collect into a large convective cluster which persists as a steady coherent structure.

In a recent paper entitled "**Convection induced by selective absorption of radiation: A laboratory model of conditional instability**", *Dyn. Atmos. Oceans-Special Issue dedicated to Dr. Pfeffer*, 27, 367-382, Dr. Krishnamurti formulates the governing equations to include the fluid-state-dependent heat source. A linear stability analysis shows that the critical Rayleigh number for onset of motion is drastically reduced. Furthermore, the cell size at onset is larger by a factor of  $\sqrt{3/2}$  than in the classical Rayleigh-Benard convection problem. However, the laboratory fluid cells were much further broadened (by a factor of eight or 10) when they penetrated into the stably stratified fluid above. Steady finite



Schematic diagram of the apparatus.

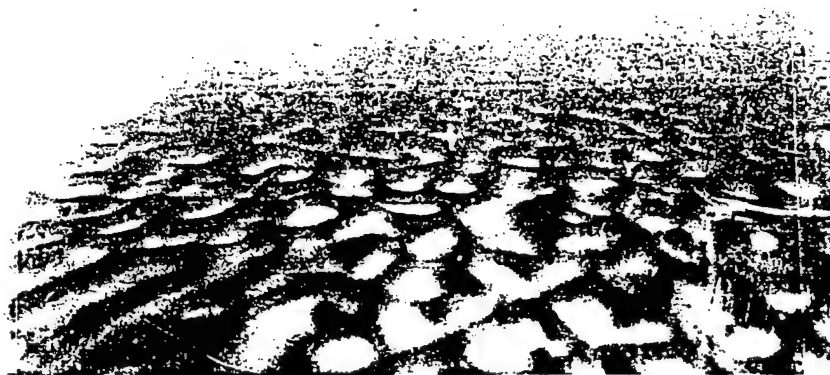
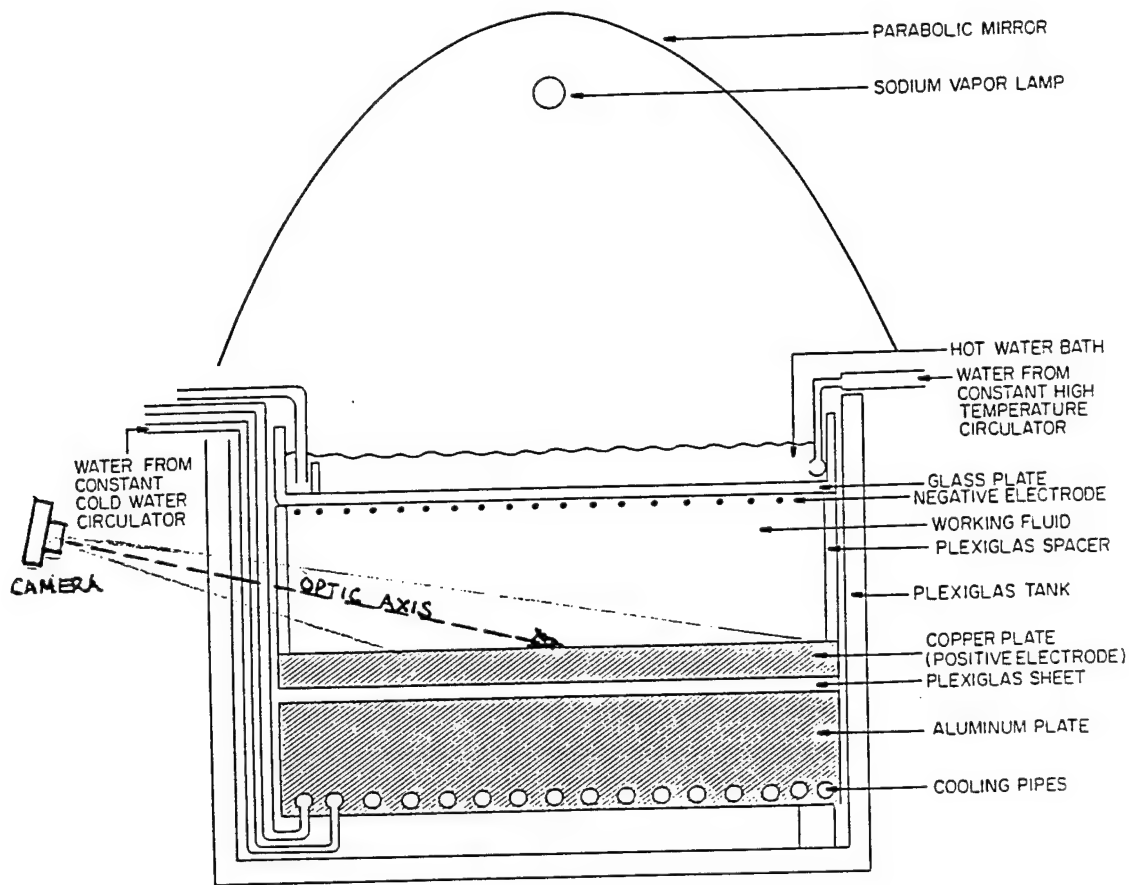


Fig. 2. View (from the side of the tank looking downward) of a stably stratified fluid with polygonal cells driven by internal heating of a shallow layer near the bottom boundary. Here  $N^2 = 3.0 \times 10^{-3}$ ,  $\gamma = 1.6$ . Overall Rayleigh number is negative. Local Rayleigh number near the bottom is approximately  $10^2$ .

amplitude solutions and their stability were analyzed and it was shown that there is a parameter range in which finite amplitude hexagonal cells are stable.

The Bjerknes "slice method" leads to the suggestion that the ratio of the cloud area  $A_c$  to the area  $A_e$  of the environment should approach zero; that is the descent should be spread over infinite area. By making  $A_c/A_e$  approach zero, one removes the need to move stable (potentially warm) air downwards. The following is an improvement on this estimate, based on our laboratory observations.

As in the slice method, we take

$$w_c A_c = w_e A_e \quad (1)$$

where  $w_c$  is the vertical velocity in the rising flow and  $w_e$  the magnitude of the vertical velocity in the sinking region. In the sinking region we take the steady heat equation as

$$w_e \partial T_e / \partial z = \kappa \partial^2 T_e / \partial z^2 \quad (2)$$

where  $\kappa$  is the thermal diffusivity and  $T_e$  is the temperature in the environment. The downward advection of warm fluid is to be balanced by heat diffusion to the cold fluid below. Since heat diffusion is a slow process it follows that  $w_e$  should be small i.e. the sinking should be spread over an area larger than  $A_c$ . *Only by having broad and thus slow sinking can the warm fluid be cooled by heat conduction to the cold bottom boundary.*

In the rising area we determine the vertical velocity  $w_c$  by balancing the buoyancy force with viscous dissipation. The buoyancy force depends upon the temperature  $T_c$  of the radiatively heated fluid over  $T_e$  derived from equation (2), which depends upon  $w_e$ . Then enforcing equation (1) we find that the ratio of horizontal length scales  $l_e / l_c$  of sinking to rising is given by

$$l_e / l_c \propto (R_c)^{1/2}$$

where  $R_c$  is the local Rayleigh number in the "cloud" region

$$R_c = g \alpha (T_c - T_e) d^3 / \kappa \nu,$$

$d$  is the depth of the radiatively heated layer,  $\nu$  is the kinematic viscosity and  $\alpha$  the thermal expansion coefficient. Calculation shows that

$$l_e / l_c \approx 10$$

in agreement with observations.

In addition to the above research, Dr. Krishnamurti has been studying double diffusive convection in laboratory experiments. The importance of determining a reliable heat flux law for salt fingers cannot be overestimated. Atmospheric behavior is affected by the ocean and vice versa. The magnitude of the poleward heat flux accomplished by the atmosphere, in particular, depends on that accomplished by the ocean. Most of the poleward heat flux in the tropics and subtropics is, in fact, carried by the ocean. An increased surface heating of the subtropical oceans might be expected to lead to an increased poleward heat transport via the thermohaline circulation. But this poleward transport is modified by vertical transport via salt-fingers. Prior flux laws have been determined from laboratory two-layer 'rundown' experiments. They give heat and salt fluxes proportional to  $R^n$ , with  $n = 1/3$ , where  $R$  is the Rayleigh number. These have been found to predict fluxes that are an order of magnitude too large in oceanic application. Some of the reasons for this inapplicability are understood. Dr. Krishnamurti's efforts are not 'rundown' experiments, but, rather, *steady-state* experiments, performed with fixed boundary values. Her experience with heat flux in Rayleigh-Benard convection, where it was classically argued that  $n = 1/3$ , has shown that  $n = .25$  or  $n = .20$ , depending on past history (hysteresis). Since Rayleigh numbers are large in nature – typically the salt Rayleigh number is of order 1010 to 1015 in the subtropical oceans – a difference in exponent of  $n = 1/3$  or  $n = 1/5$  could easily account for two orders of magnitude difference in flux. Although these steady state experiments are difficult and time-consuming, we feel it vital to know an accurate flux law. Accordingly, Dr. Krishnamurti has constructed and has in operation a new salt-finger convection apparatus. in which the boundary conditions are *maintained*.

Three significant advances made by Dr. Krishnamurti and her collaborators under AFOSR Grant F49620-96-1-0172 are

(i) The experimental finding that wide salt fingers (as wide as they are tall) occur near the onset of convection, as predicted theoretically by Baines and Gill, *J. Fluid Mech.*, 1969). Previously, salt fingers have always been seen to be long and narrow (usually millimeters in width and as much as meters in length). This experimental result was found by Dr. Krishnamurti's student, Mr. Y. K. Jo. These experiments are difficult to design and execute, which explains why several investigators over the past three decades have failed. Although his results are late in coming, they verify one of the few theoretical results and thereby add strength to the theory.

(ii) Improvement of the visualization aspects of this work, with the construction of a 4 ft. by 8 ft. by 10 ft. tall 'camera' which contains the convection apparatus.

(iii) In a 6-foot deep fluid Professor Krishnamurti found alternating salt finger zones and convection zones, which have not been reported previously in the published literature. What is more remarkable is that the salt fingers are not upright, but instead tilt in such a way as to transport horizontal momentum, as well as salt, vertically, via Reynolds stresses. By so doing, they impart a large-scale flow superimposed on the convection in the convection zones. A paper entitled "**Salt finger experiments with maintained boundary conditions**", which deals with the interpretation of her experimental results, is in preparation jointly with Professor George Veronis of Yale University.

## 5. Numerical Methods

Under AFOSR Grant F49620-96-1-0172, Professor Navon and student Zhijin Li made an extensive study of adaptive finite element methods (FEM), especially h-p adaptive finite element methods (FEM) for solving partial differential equations. This was done in connection with Mr. Li's doctoral research entitled **"Finite Element Applications and analysis for Singularly Perturbed Problems and shallow Water Equations"**. In this work, they investigated the use of FEM for singularly perturbed problems (SPP). It is known that the classical FEM cannot achieve global uniform convergence for general SPP, where the error estimate is independent of the perturbation parameter. In their work, they proposed a systematic technique for constructing global uniformly convergent FEM schemes on some piecewise uniform meshes to solve singularly perturbed elliptic problems in two space dimensions. Four model problems were considered; i.e., an anisotropic model, the reaction-diffusion model, the convection-diffusion model and a two-parameter model. Extensive numerical results were obtained which supported their theoretical analysis. In addition, two papers related to the use of a particular adaptive FEM for solving sharp boundary layers have been written describing new research results obtained using the adaptive Shishkin mesh.

A paper entitled **"Uniformly Convergent Finite Element Methods For Singularly Perturbed Elliptic Boundary Value Problems I: Reaction-Diffusion Type"** by Dr. Navon and Mr Li has been accepted for publication by the journal *Computers and Mathematics with Applications*. In this paper they consider the bilinear finite element method on a Shishkin mesh for the singularly perturbed elliptic boundary value problem  $-\epsilon^2 (\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2) + \alpha(x,y) u = f(x,y)$  in two space dimensions. By using a very sophisticated asymptotic expansion, they prove that their method achieves almost second-order uniform convergence rate in  $L^2$ -norm. Extensive numerical results confirm their theoretical analysis.

Another paper entitled **"Uniformly Convergent Finite Element Methods For Singularly Perturbed Elliptic Boundary Value Problems II: Convection-Diffusion Type"** has been submitted to the Journal *Computer Methods in Applied Mechanics and Engineering* and is now under review. In this paper they consider the standard bilinear finite element method (FEM) and the corresponding streamline diffusion FEM for the singularly perturbed elliptic boundary value problem  $-\epsilon^\alpha (\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2) + b(x,y) c \cdot \nabla u + a^\alpha(x,y) u = f(x,y)$  in the two space dimensions. By using the asymptotic expansion method of Vishik and Lyusternik and the technique they used in the previous paper, they prove that the standard bilinear FEM on a Shishkin type mesh achieves first-order uniform convergence rate globally in  $L^2$ -norm for both the ordinary exponential boundary layer case and the parabolic boundary layer case. Extensive numerical results have been carried out for both cases. The numerical results show that the methods they developed perform much better than either the classical standard or streamline diffusion FEM.

More recently Mr Chris Homescu, a Ph. D candidate jointly supervised by Dr. Navon and Dr. Youssef Hussaini has begun research on the optimal control of the Navier-Stokes equations in 2-D and 3-D. Issues of optimal control in the presence of discontinuities are being

given particular attention. Mr. Homescu will defend his prospectus at the end of the 1998 Fall semester.

## 6. Technology Transitions

Technology transitions this year have again been a two-way street. Drs. Barcion and Pfeffer have received much help from ASU Professors Nicolaenco and Mahalov in learning to apply their methodology for obtaining equations for slow time scales to new problems, and have provided them with much information and insight into the relevant meteorological problems to be tackled in the future. This has been done both through one-on-one discussions and also by providing them with extensive written material including mathematical derivations and interpretations of the derived equations.

Professor Navon is also transferring both theoretical and numerical results on singularly perturbed elliptical problems involving sharp boundary layers and their numerical solution to the research group headed by Professor Owe Axelsson at the University of Nijmegen in the Netherlands. Dr. Nijmegen visited FSU in June of this year to develop a further collaboration with Dr. Navon and his student, Zhijin Li.

Professor Ruby Krishnamurti has transferred her mathematical model to FSU Geology Professor David Furbish and is preparing an experiment to collaborate with Dr. Furbish on the geology of sinkholes. Her model allows acidic water to seep through porous limestone. The stability theory predicts the spacing of sinkholes that should develop. Moreover, she has transferred to Dr. Furbish the details of the method developed by Krishnamurti and Zhu (1996) for measurement of momentum transport to be used to measure the effect of sand dunes and ripples (which are a self induced roughening of an interface) upon the vertical transport of horizontal momentum.

## 7. Awards Since 1 October 1996

In April 1997, Dr. Pfeffer was named "Distinguished Research Professor" by Florida State University.

A special 743-page issue of the scientific journal *Dynamics of Atmospheres and Oceans*, published in early 1998 was dedicated to the scientific and administrative accomplishments of Professor Pfeffer. Forty four joint and individual papers for the special issue were submitted by 90 scholars from major universities and research centers in the U. S. and abroad (from England to China), including Fellows of the British Royal Society and a member of the U.S. National Academy of Sciences. Included are scientific papers by scholars from Harvard, Yale, The University of Chicago, UCLA, UC Davis, Johns Hopkins University, Georgia Tech, the Universities of Colorado, Hawaii and Miami, Arizona State, Mississippi State, Iowa State, FSU, NOAA, NASA and NORDA in the United States, and from Oxford and Reading Universities in the UK, the Max Plank University in Germany, the Ecole Normal Superior in Paris, France, the University of Copenhagen in Denmark, the Academia Sinica in China, the

European Center for Medium Range Weather Forecasting in the UK and the Bedford Institute of Oceanography in Canada. The subject matter of the papers includes fundamental atmospheric and ocean dynamics, atmospheric and ocean data assimilation and analysis, mathematical and computer techniques and a discussion of the impact of Dr. Pfeffer's teaching on his students. The issue will appear in the Fall of 1997.

In August 1996, Dr. Ruby Krishnamurti was invited to speak on "Low frequency oscillations in turbulent convection" at Stockholm University's Department of Meteorology. In January 1997 she was invited to speak on "Salt, heat and momentum transport by salt fingers" and on "thermal oscillators" at the Nonlinear Dynamical Systems Workshop held at Florida State University's Supercomputer Computations Research Institute.

## **8. Publications under AFOSR Grant F49620-96-1-0172**

Barcilon, A. and C. H. Bishop, 1998: Non-modal development of baroclinic waves undergoing horizontal shear deformation. *J. Atmos. Sci.*, in press

Barcilon, A. and P. G. Drazin, 1998: "Nonlinear waves of vorticity," to be submitted to *J. Fluid Dynamics*.

W. Blumen and A. Barcilon, 1998: Application of Bretherton's interpretation of baroclinic instability in the presence of horizontal shear and compressibility. *Dyn. Atmos. Oceans-Special Issue dedicated to Dr. Pfeffer*, **27**, 151-160.

W. Blumen and A. Barcilon, 1998: An analysis of the constant shear quasigeostrophic baroclinic-barotropic instability problem. To be submitted to *J. Atmos. Sci.*, Aug. 1997.

Casey, M., T. R. Nathan, and A. Barcilon, 1998: Effects of seasonal thermal forcing on atmospheric low-frequency variability. To be submitted to *J. Atmos. Sci.*

DeFonso, E., and T. R. Nathan, 1998: Examination of the connection between the Pacific and Atlantic storm tracks using an idealized nonlinear model. To be submitted to *J. Atmos. Sci.*

de Pondaca, M. S. F. V., A. Barcilon and X. Zou, 1998: Non-modal development of baroclinic waves undergoing horizontal shear deformation. *J. Atmos. Sci.*, in press.

de Pondaca, M. S. F. V., A. Barcilon and X. Zou, 1998: An adjoint sensitivity study of the efficacy of modal and non-modal perturbations in causing model block onset. *J. Atmos. Sci.*, in press.

Krishnamurti, R. and Y. Zhu, 1996, Heat and momentum transport in sheared Rayleigh-Benard convection, *Physica D.*, **97**, 126-132.



- Krishnamurti, R., 1998, Convection induced by selective absorption of radiation: A laboratory model of conditional instability, *Dyn. Atmos. Oceans—Special Issue dedicated to Dr. Pfeffer*, **27**, 367–382.
- Krishnamurti, R., 1998, Low frequency oscillations in turbulent convection, *Proc. Indian Academy of Sci.*, in press.
- Krishnamurti, R. and G. Veronis, 1997, Salt finger experiments with maintained boundary conditions, In preparation.
- Li, Long, and T. R. Nathan, 1996: Effects of low frequency tropical forcing on intraseasonal tropical–extratropical interactions. *J. Atmos. Sci.*, **54**, 332–346.
- Li, Zhijin and I.M. Navon, 1997: Uniformly convergent finite element methods for singularly perturbed elliptic boundary value problems I: Reaction–Diffusion Type. Accepted for publication in *Computers and Mathematics with Applications*.
- Li, Zhijin and I.M. Navon, 1997 : Uniformly convergent finite element methods for singularly perturbed elliptic boundary value problems ii: convection–diffusion type. Submitted for publication to *Computer Methods in Applied Mechanics and Engineering*.
- Li Zhijin, A. Barcilon and I.M. Navon, 1998: Study of Block Onset Using Sensitivity Perturbations in Climatological Flows. *Mo. Wea. Rev.*, submitted Jan. 1998.
- Nathan, T. R., 1998: Nonlinear spatial instability in zonally varying baroclinic flow. *Dyn. Atmos. Oceans—Special Issue dedicated to Dr. Pfeffer*, **27**, 81–90.
- Parlange, M., and T. R. Nathan, 1996: Forced baroclinic wave dynamics at minimum critical shear: potential vorticity homogenization, vacillation and equilibration. *J. Atmos. Sci.*, **53**, 3490–3502.
- Shellito, L., T. R. Nathan, and L. Li, 1998: Effects of El Nino on the propagation of energy from low frequency tropical forcing to midlatitudes. Submitted to *J. Atmos. Sci.*
- Wang, B., A. Barcilon and Z. Fang, 1997: Stochastic dynamics of El Nino–southern oscillations. To be submitted to *J. Atmos. Sci.*, July 1997.